Morphogenesis and adjustment of flat rod structures.

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ABSTRACT. The mathematical apparatus allowing receiving the preliminary form of rod construction as well as changing positions of nods of the received model, systemically redistributing internal forces in its rods was considered in this paper. To achieve this, the parametric equations of rods of the construction are used. These equations reflect the relationship between coordinates of the model, stiffness parameters, and the characteristics of the external forces acting on the structure. The flat hinged trusses, the rods of which work only on tension or compression, are considered as an object of researches. Several examples of the shaping and adjustment of flat trusses are given, taking into account different initial conditions.

1. Introduction

The process of designing the majority of architectural objects consists of two principal stages. The first stage is the development of a general concept, form and spatial planning decisions in accordance with the purpose of the building or structure. The second stage is the development of constructive solutions and engineering systems. At the first stage, the architects are responsible for the design process, on the second stage – construction engineers who ensure the stability and structural strength of the constructions taking into account the forms accepted by the architect. The architects in their work use the rules of sketch pictures constructing and sometimes morphogenesis methods, taking into account the normative requirements for interior layouts. In their turn, construction engineers use analytical calculation methods and numerical simulation methods to determine the components of the stress-strain state of structures. If the shape of construction is
unstable or not sufficiently reliable, then the concept returns for improvement to architects. This process must be repeated again and again until it is picked stable structural configuration.

At the same time, there is no common method that would simultaneously allow solving the problems of the shaping (morphogenesis) of structures and determining the internal forces in them. Along with this, the creation and development of such methods is very actual and can allow significantly reducing waste of time to all those who are involved in the design process, especially when it comes to frame constructions and rod structures of coatings. Let's consider one of such methods on an example of flat rafter trusses.

2. Mathematical model of rod structures

Often, when we design large-span buildings or buildings with a complicated sloped or curved shape of the roof, we use hinged trusses and other rod constructions as supporting rafter structures [1, 2]. In some cases, we design cable-stayed structures [3]. The specificity of the operation of such structures is that there are no bending moments in their rods (links). These elements work only on compression or stretching, and therefore the character of their work is relatively easy to predict. In addition, the absence of bending moments increases the reliability of the structure and simplifies the process of their calculation.

In this paper, we will consider the operation of rods of construction only in the framework of elastic deformations and the preservation of their stability.

2.1. Static Equilibrium and Morphogenesis of Structures

In works [4, 5, 6] was demonstrated an approach to shaping of discrete geometric models based on the concept of multi-link elastic mesh structures. The process of their shaping is identical to the rod constructions morphogenesis and represents the determining of coordinates of free nodes, when there are known its topological features (the order of connections and supporting nodes), the nodal loads and conditional stiffness parameters $N_{a,b}$, that should be set for the start as a constant ratios of internal efforts in rods $R_{a,b}$ to their lengths $\delta_{a,b}$:

$$ N_{a,b} = N_{b,a} = R_{a,b} / \delta_{a,b}. \quad (2.1) $$

A system of equilibrium equations that describes the weighted state of some $a^{th}$ node of a two-dimensional model can be written in this form [6]:
\[
\sum_{i=1}^{m} (s_i - s_a) \cdot N_{a,i} + \mathfrak{I}_{a} = 0, \text{ or: }
\]
\[
\sum_{i=1}^{m} [N_{a,i} \cdot s_i] - s_a \cdot \sum_{i=1}^{m} N_{a,i} + \mathfrak{I}_{a} = 0,
\]

where: \( s \) – generalized coordinate designation (x and y); \( \mathfrak{I}_{a} \) – projections of vector of the field of influence \( \mathfrak{I} \) in \( a^{th} \) node (\( \mathfrak{I}_{x,a} \) and \( \mathfrak{I}_{y,a} \)); \( n \) – amount of nodes, adjacent to \( a^{th} \) node. System (2.2) can be represented for more clarity in the form of a computational template (Figure 1).

![Computational Template](image)

Fig. 1. Computational template equivalent to the system (2.2)

Assigning the coordinates of the support nodes as initial parameters, the values of the external node loads \( \mathfrak{I}_{a} \) and conditional stiffness parameters \( N_{a,b} \), and then constituting a system of equations of the type (2.2), and resolving it relative to the coordinates of free nodes, we obtain the initial form of the construction. If the parameter \( N_{a,b} \) has a positive sign – then rod \( S_aS_b \) will work on stretching, but if negative – then on compression.

Thus, the process of model formation consists in finding the coordinates of its nodes, taking into account the predefined nature of the operation of the rods. However, the received form does not always satisfy the requirements of architects and designers (construction engineers), as it may not fit into the overall concept of the building or require consumption of larger than expected quantities of building materials.

2.2. Correction of the Shape of Rod Structure

To obtain the desired shape of the projected construction, it is necessary to clarify the positions of individual or all of its nodes. However, when the positions of the nodes are forced to change, conditional stiffness parameters of the rods \( N_{a,b} \) and values of their internal efforts \( R_{a,b} \) will be unknown, and their search in most cases will require using of numerical methods of calculation.
Therefore, it is better to go from the reverse and try to pick up such stiffness parameters (and such distribution of internal forces) that will lead to the desired positions of free nodes of construction. If the construction contains a large number of rods, then it is simply impossible to manually select the required stiffness parameter’s configuration. In this case, it is necessary to resort to a system mathematical solution, compiling and solving with respect to unknown conditional stiffness parameters the corresponding number of equations. The parametric equations of the state of rods proposed in [7, 8] can serve as such equations. In the most universal form, these equations can be written as follows:

1) for rods, connecting two free nodes of a construction (S<sub>a</sub> and S<sub>b</sub>):

\[
\sum_{i=1}^{m-1} \delta_{a,i}^2 \cdot N_{a,i} + \chi \cdot \delta_{a,b}^2 \cdot N_{a,b} \cdot \sum_{j=1}^{n-1} \delta_{b,j}^2 \cdot N_{b,j} - (\varphi_a + \varphi_b) + B_{a,b} = 0 ,
\]

(2.3)

2) for rods, connecting one free and one fixed (basic) nodes (S<sub>a</sub> and S<sub>fix</sub>):

\[
\sum_{i=1}^{m-1} \delta_{a,i}^2 \cdot N_{a,i} + \chi \cdot \delta_{a,fix}^2 \cdot N_{a,fix} - \varphi_a + \left( R_{x,fix} \cdot x_{fix} + R_{y,fix} \cdot y_{fix} \right) + B_{a,fix} = 0 ,
\]

(2.4)

where: m and n – numbers of nodes, adjacent to nodes S<sub>a</sub> and S<sub>b</sub> (or to node S<sub>fix</sub>); \( \chi \) – some nonnegative constant; \( \varphi_a \) and \( \varphi_b \) – nodal values of the scalar potentials (fields of the objective function); \( R_{fix} \) – force values in rods, which are connected with the hinged supports in the nodes S<sub>fix</sub>; \( B_{a,b} \) and \( B_{a,fix} \) – common operational constants of integration.

Equations (2.3) and (2.4) allow you to link the values of the coordinates of the nodes (x and y), conditional stiffness parameters of rods \( \chi \) and values of nodal potentials \( \varphi \). These equations as well as equation (2.2), can be represented visually (graphically) in the form of computational templates intended for superimposing on the design schemes of constructions and for formulation of the equations themselves. Such templates are shown at the Figures 2 and 3.

The process of correction must be realized by achieving the specified values of the objective function (target functions) in the free nodes of the model. The target functions correspond to the functions of the potential \( \varphi \) at the nodes.

Besides, the adjusting process can consist in synthesizing the shape of a truss with such optimal parameters, in which its weight or volume will be minimal (as in [14-17]), while maintaining the stability of the structure and its work in the area of elastic deformations. In this case, the objective function must take into account the strength characteristics of the rods and used amount of material.

The entire process is an iterative calculus.

After each iteration, the current values of the node potentials \( \varphi \) must be updated, and
compared with the expected values $\varphi'$. And if the difference $\Delta \varphi$ between current and expected values exceeds the specified error value $\varepsilon$, then the calculation must be repeated taking into account the replacement of $\varphi$ on $\varphi'$.

$$-(\varphi_a + \varphi_b) + B_{a,b} = 0$$

Fig. 2. Computational template equivalent to the equation (2.3)

$$-\varphi_a + \left(R_{x,fix} \cdot x_{fix} + R_{y,fix} \cdot y_{fix}\right) + B_{a,fix} = 0$$

Fig. 3. Computational template equivalent to the equation (2.4)

So, basing on equations (2.2) – (2.4), in the matrix form, the process of morphogenesis and further adjusting the shape of the rod structure can be described by the following system solution:
\[
[s^p] = [N^{p-1}]^{-1} \cdot \left( -[g^{p-1}] - [\zeta^p] \right),
\]  
(2.5)

\[
\{N^p\} = \left( [\delta^p]^2 \right)^{-1} \cdot \left( \{\varphi'^p\} - \{\varphi^p\} + \left( [\delta^p]^2 \right) \{N^{p-1}\} \right).
\]  
(2.6)

Here: \([s]\) – matrix of coordinates (with dimension \(k \times 2\), where \(k\) – amount of free nodes of the system); \([g]\) – matrix of boundary conditions (with dimension \(k \times 2\)); \([\zeta]\) – matrix of external influences (with dimension \(k \times 2\)); \([N]\) – matrix of conditional stiffness parameters of rod structure (with dimension \(k \times k\)); \(\{N\}\) – vector-column of conditional stiffness parameters of the rod structure; \([\delta^2]\) – matrix of geometrical parameters of rod structure (with dimension \(h \times h\), where \(h\) – number of rods of the model); \(\{\varphi\}\) – vector-column of nodal values of scalar potential; \(\{\varphi'^p\}\) – vector-column of expected nodal values of scalar potential; \(p\) – index corresponding to the current step of the iterative calculus.

Using the expression (2.5), the coordinates of nodes at the current stage of the iterative calculus can be calculated. The expression (2.6) – reflects the principle of correcting all stiffness parameters of rods of the construction.

3. Increasing the efficiency of the flat truss shape adjusting process

To reduce the number of iteration cycles of the solution of system (2.5) – (2.6), it is necessary to select the optimal objective function \(\varphi\). In the case of adjusting the position of the model nodes by moving to specific points, the task is reduced to a step-by-step minimization of the distances between the current and expected coordinates. Therefore, as an objective function of each free node \(S_a\) should be chosen a length between its current \((x_a, y_b)\) and established \((x_a', y_b')\) position:

\[
\varphi_a = \varphi(s_a) = \zeta(x_a, y_b) = \eta \cdot \left( (x'_a - x_a)^2 + (y'_a - y_a)^2 \right)^{1/2},
\]  
(3.1)

where \(\eta\) – coefficient introduced for influencing the speed of convergence of the iterative calculation (for goals of mechanic it should be measured in \(kN\)). In this case, the expected value of a nodal potential will be equal to zero: \(\varphi'_a = 0\) (\(kN \cdot m\)).

However, as mentioned in [9], if we choose distances of the type (3.1) as target functions, then in the case of an unfortunate choice of the coefficient \(\eta\) and when the calculation error \(\varepsilon\) is low, the iterative calculus may not converge at all. To avoid this, it is necessary to use not only the constant \(\eta\), but also the additional logical operator \(\mathfrak{9}\). In this case, this operator should be applied not to the nodal potentials, but to the values of the total rod potentials contained in the vector-column \(\{\varphi\}\). For an arbitrary rod \(S_iS_j\) (at the \(p^{th}\) stage of the iterative calculus), the operator \(\mathfrak{9}^p_{ij}\) should have the following form:
\[ \Theta_{i,j} = \zeta(\Delta \varphi_{i,j}) = \begin{cases} 1 & \text{if } \Delta \varphi_{i,j} > 0, \\ 0 & \text{if } \Delta \varphi_{i,j} = 0, \\ -1 & \text{if } \Delta \varphi_{i,j} < 0, \end{cases} \tag{3.2} \]

where:

\[ \Delta \varphi_{i,j} = \varphi_{i,j}^p - \varphi_{i,j}^{p-1}, \tag{3.3} \]

\[ \varphi_{i,j}^p = \begin{cases} (\varphi_i^p + \varphi_j^p) & \text{if } S_i S_j \equiv S_a S_b, \\ \varphi_i^p & \text{if } S_i S_j \equiv S_a \text{fix}, \\ \varphi_j^p & \text{if } S_i S_j \equiv S_b \text{fix}. \end{cases} \tag{3.4} \]

The expression (3.2) can be replaced by the hyperbolic tangent function:

\[ \Theta_{i,j} = \zeta(\Delta \varphi_{i,j}) = \tanh \left( \frac{\alpha \cdot \Delta \varphi_{i,j}}{2} \right) = \frac{1 - \exp(-\alpha \cdot \Delta \varphi_{i,j})}{1 + \exp(-\alpha \cdot \Delta \varphi_{i,j})}. \tag{3.5} \]

where \( \alpha \) is a coefficient, the value of which determines the sharpness of the change in the character of the function (3.5) in the transition from \(-1\) to \(+1\).

In addition, the model correction using the system (2.5) – (2.6), is partially similar to the algorithm for selecting the weighting coefficients in training of neural networks [10-12]. As a result, the more accurate the data on the final discrete form of the structure, the higher the probability of obtaining a solution of the task with a given accuracy. In the system (2.6), the data on the finite form of the model are represented as a vector of expected values of the potential \( \{ \varphi \} \). However, this may not be enough, since if not all nodal coordinates are corrected, but only some of them, then there are an infinite number of suitable locations for other free nodes. As a result, the iterative computation can be significantly delayed in time or never be completed at all with a given accuracy, since in some cases the system will not be able to determine independently which variant of the location of not investigated free nodes is optimal.

Therefore, it is proposed to contribute into the system (2.6) one more set of accurate data on the expected state of the construction. These are data on the geometric parameters of those fragments of the construction, the shape of which is predetermined within the input conditions of the task. The corresponding data are contained in the matrix \( [\delta^2] \), which is already existing in the system (2.6). This matrix contains the squares of the lengths of all the rods of the model and has the form:

\[ [\delta^2] = \begin{bmatrix} \chi \cdot \delta_{1,1,1}^2 & \delta_{1,1,2}^2 & \cdots & \delta_{1,1,h}^2 \\ \delta_{1,2,1}^2 & \chi \cdot \delta_{1,2,2}^2 & \cdots & \delta_{1,2,h}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{h,1,1}^2 & \delta_{h,1,2}^2 & \cdots & \chi \cdot \delta_{h,h}^2 \end{bmatrix}. \tag{3.6} \]

Here the diagonal elements contain products of constants \( \chi \) and squares of bond lengths for
which an equations of the type (2.6) corresponding to a particular row of the matrix is composed; the remaining elements include either the squares of the lengths of the bonds (rods) corresponding to the given cell of this matrix, or zeros. The symbol "\lor" means the operation "or".

In the matrix \[ \delta^2 \] we replace the lengths of the rods \( \delta \) in the current stage of the computations by the expected lengths \( \delta' \) for those rods whose position of nodes is uniquely determined. Such a matrix will have the following form:

\[
\begin{bmatrix}
\chi \cdot (\delta^2_{i,j,1,1} \lor \delta^2_{i,j,1,2}) & (\delta^2_{i,j,1,1} \lor \delta^2_{i,j,1,2}) \lor 0 & \cdots & (\delta^2_{i,j,1,h} \lor \delta^2_{i,j,1,1}) \lor 0 \\
(\delta^2_{i,j,2,1} \lor \delta^2_{i,j,2,2}) \lor 0 & \chi \cdot (\delta^2_{i,j,2,2} \lor \delta^2_{i,j,2,2}) & \cdots & (\delta^2_{i,j,2,h} \lor \delta^2_{i,j,2,2}) \lor 0 \\
\vdots & \vdots & \ddots & \vdots \\
(\delta^2_{i,j,h,1} \lor \delta^2_{i,j,h,1}) \lor 0 & (\delta^2_{i,j,h,2} \lor \delta^2_{i,j,h,2}) \lor 0 & \cdots & \chi \cdot (\delta^2_{i,j,h,h} \lor \delta^2_{i,j,h,h}) \\
\end{bmatrix}
\] (3.7)

To understand how to integrate the matrix \[ \delta^2 \] into the solution (2.6), it is necessary to analyse the process of its formation. Expression (2.6) is a solution of a system of equations of the types (2.3) and (2.4) [13]. Such a system in the matrix form for the previous \((p-1)\)th and current \(p\)th stages of the iterative calculus can be written as:

\[
\begin{bmatrix}
\delta^{(p-1)2} \\
\delta^p \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{N}^{p-1} \\
\mathbf{N}^p \\
\end{bmatrix} - \begin{bmatrix}
\varphi^{p-1} \\
\varphi^p \\
\end{bmatrix} + \begin{bmatrix}
\mathbf{B}^{p-1} \\
\mathbf{B}^p \\
\end{bmatrix} = 0,
\]

(3.8)

\[
\begin{bmatrix}
\delta^p \\
\delta^{p2} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{N}^p \\
\mathbf{N}^{p-1} \\
\end{bmatrix} - \begin{bmatrix}
\varphi^p \\
\varphi^{p-1} \\
\end{bmatrix} = 0.
\]

(3.9)

In order to express the conditional stiffness parameter vector \( \{\mathbf{N}^p\} \) at the current simulation stage, we need to express the vectors of the constants \( \{\mathbf{B}^{p-1}\} \) and \( \{\mathbf{B}^p\} \) from equations (3.8) and (3.9), equating them. At the current stage, it is necessary to replace the potential vector \( \{\varphi^p\} \) and the matrix of geometric parameters \( \{\delta^p\} \) by the expected vector \( \{\varphi^p\} \) and the matrix \( \{\delta^p\} \):

\[
\begin{bmatrix}
\delta^{(p-1)2} \\
\delta^p \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{N}^{p-1} \\
\mathbf{N}^p \\
\end{bmatrix} - \begin{bmatrix}
\varphi^{p-1} \\
\varphi^p \\
\end{bmatrix} = \begin{bmatrix}
\delta^p \\
\delta^{p2} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{N}^p \\
\mathbf{N}^{p-1} \\
\end{bmatrix} - \begin{bmatrix}
\varphi^p \\
\varphi^{p-1} \\
\end{bmatrix}.
\]

(3.10)

Moreover, if we take into account that the potential vectors and the matrix of geometric parameters in the current and previous stage of the calculus remain equal until the moment of substitution of the expected parameters, that is:

\[
\begin{bmatrix}
\delta^{(p-1)2} \\
\delta^p \\
\end{bmatrix} = \begin{bmatrix}
\delta^p \\
\delta^{p2} \\
\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}
\varphi^{p-1} \\
\varphi^p \\
\end{bmatrix} = \begin{bmatrix}
\varphi^p \\
\varphi^{p-1} \\
\end{bmatrix},
\]

then the equation (3.10) takes the following form:

\[
\begin{bmatrix}
\delta^p \\
\delta^{p2} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{N}^p \\
\mathbf{N}^{p-1} \\
\end{bmatrix} - \begin{bmatrix}
\varphi^p \\
\varphi^p \\
\end{bmatrix} = \begin{bmatrix}
\delta^p \\
\delta^{p2} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{N}^{p-1} \\
\mathbf{N}^p \\
\end{bmatrix} - \begin{bmatrix}
\varphi^p \\
\varphi^{p-1} \\
\end{bmatrix}, \text{ from which we obtain:}
\]

\[
\mathbf{N}^p = \begin{bmatrix}
\delta^p \\
\delta^{p2} \\
\end{bmatrix}^{-1} \cdot \left(\begin{bmatrix}
\varphi^p \\
\varphi^p \\
\end{bmatrix} - \begin{bmatrix}
\delta^p \\
\delta^{p2} \\
\end{bmatrix} \begin{bmatrix}
\mathbf{N}^{p-1} \\
\mathbf{N}^p \\
\end{bmatrix}\right). \]

(3.11)

Exactly this solution should be used instead of the formula (2.6) to correct the stiffness parameters.
4. Examples of shape formation and truss calculation

Let us consider several examples of the formation of an elementary flat truss taking into account different initial conditions. Let the truss consist of two fixed hinged-immovable and three free joints hinged by unit forces, connected by seven rods, as shown in Figure 4. In addition, we sequentially number the rods for further clarity of the Figures.

![Flat truss model with three free and two fixed nodes](image)

Fig. 4. Flat truss model with three free and two fixed nodes (truss is formed and adjusted in accordance with the conditions No. 1 and No. 2 of Table 1)

To begin with, we will compose the equations of the equilibrium state of free nodes. These equations of type (2.2) will have the following appearance for nodes $S_2, S_3$ and $S_4$:

$$
\begin{align*}
- \left( N_{1,2} + N_{2,3} + N_{2,4} \right) s_2 + N_{1,2} : s_1 + N_{2,3} : s_3 + N_{2,4} : s_4 + F_{x_2} &= 0, \\
- \left( N_{1,3} + N_{3,4} + N_{4,5} \right) s_3 + N_{1,3} : s_1 + N_{3,4} : s_4 + N_{4,5} : s_5 + F_{x_3} &= 0, \\
- \left( N_{2,4} + N_{4,5} + N_{5,3} \right) s_4 + N_{2,4} : s_2 + N_{4,5} : s_5 + F_{x_4} &= 0.
\end{align*}
$$

(4.1)

(4.2)

(4.3)

Now we write parametric equations of the type (2.3) and (2.4) for rods $S_1S_2, S_1S_3, S_2S_3, S_2S_4, S_3S_4, S_3S_5$ and $S_4S_5$ respectively:

$$
\begin{align*}
\chi : \delta_{1,2}^2 \cdot N_{1,2} + \delta_{2,3}^2 \cdot N_{2,3} + \delta_{2,4}^2 \cdot N_{2,4} - \varphi_2 &= \left( R_{x_1} \cdot x_1 + R_{y_1} \cdot y_1 \right) + B_{1,2} = 0, \\
\chi : \delta_{1,3}^2 \cdot N_{1,3} + \delta_{2,3}^2 \cdot N_{2,3} + \delta_{3,4}^2 \cdot N_{3,4} + \delta_{3,5}^2 \cdot N_{3,5} - \varphi_3 &= \left( R_{x_1} \cdot x_1 + R_{y_1} \cdot y_1 \right) + B_{1,3} = 0, \\
\chi : \delta_{2,3}^2 \cdot N_{2,3} + \delta_{2,4}^2 \cdot N_{2,4} + \delta_{1,3}^2 \cdot N_{1,3} + \delta_{3,4}^2 \cdot N_{3,4} - \left( \varphi_2 + \varphi_3 \right) &= B_{2,3} = 0, \\
\chi : \delta_{2,4}^2 \cdot N_{2,4} + \delta_{1,2}^2 \cdot N_{1,2} + \delta_{2,3}^2 \cdot N_{2,3} + \delta_{3,4}^2 \cdot N_{3,4} + \delta_{4,5}^2 \cdot N_{4,5} - \left( \varphi_2 + \varphi_4 \right) &= B_{2,4} = 0, \\
\chi : \delta_{3,4}^2 \cdot N_{3,4} + \delta_{2,4}^2 \cdot N_{2,4} + \delta_{4,5}^2 \cdot N_{4,5} + \delta_{2,3}^2 \cdot N_{2,3} + \delta_{3,5}^2 \cdot N_{3,5} - \left( \varphi_3 + \varphi_4 \right) &= B_{3,4} = 0, \\
\chi : \delta_{3,5}^2 \cdot N_{3,5} + \delta_{1,3}^2 \cdot N_{1,3} + \delta_{2,3}^2 \cdot N_{2,3} + \delta_{3,4}^2 \cdot N_{3,4} - \varphi_3 &= \left( R_{x_5} \cdot x_5 + R_{y_5} \cdot y_5 \right) + B_{3,5} = 0, \\
\chi : \delta_{5,3}^2 \cdot N_{5,3} + \delta_{1,3}^2 \cdot N_{1,3} + \delta_{2,3}^2 \cdot N_{2,3} + \delta_{3,4}^2 \cdot N_{3,4} - \varphi_3 &= \left( R_{x_5} \cdot x_5 + R_{y_5} \cdot y_5 \right) + B_{3,5} = 0.
\end{align*}
$$

(4.4)

(4.5)

(4.6)

(4.7)

(4.8)

(4.9)
\[ \chi \cdot \delta_{4,5}^2 \cdot \mathcal{N}_{4,5} + \delta_{2,4}^2 \cdot \mathcal{N}_{2,4} + \delta_{3,4}^2 \cdot \mathcal{N}_{3,4} - \varphi_4 + \left( R_{x_5} \cdot x_5 + R_{y_5} \cdot y_5 \right) + B_{4,5} = 0. \]  

(4.10)

Systems (4.1) – (4.3) и (4.4) – (4.10) are represented in the form of computational templates in Figures 5-7 и 8-14 respectively.

Fig. 5 Computational template, which is equivalent to the equations (4.1)

Fig. 6. Computational template, which is equivalent to the equations (4.2)

Fig. 7. Computational template, which is equivalent to the equations (4.3)

Fig. 8. Computational template, which is equivalent to the equation (4.4)
Fig. 9. Computational template, which is equivalent to the equation (4.5)

\[ + (R_{x_1} \cdot x_1 + R_{y_1} \cdot y_1) - \varphi_1 + B_{1,3} = 0 \]

Fig. 10. Computational template, which is equivalent to the equation (4.6)

\[ - (\varphi_2 + \varphi_3) + B_{2,3} = 0 \]

Fig. 11. Computational template, which is equivalent to the equation (4.7)

\[ - (\varphi_2 + \varphi_4) + B_{2,4} = 0 \]

Fig. 12. Computational template, which is equivalent to the equation (4.8)

\[ - (\varphi_3 + \varphi_4) + B_{3,4} = 0 \]
Fig. 13. Computational template, which is equivalent to the equation (4.9)

\[ + (R_x \cdot x_5 + R_y \cdot y_5) - \varphi_4 + B_{4,5} = 0 \]

Fig. 14. Computational template, which is equivalent to the equation (4.10)

\[ + (R_x \cdot x_5 + R_y \cdot y_5) - \varphi_4 + B_{4,5} = 0 \]

Using systems (4.1) – (4.3) and (4.4) – (4.10), the components of expressions (2.5) and (3.11) can be constructed. Let’s write them down below.

The matrix \([s^p]\) will have the form:

\[
[s^p] = [X^p \quad Y^p],
\]

where \(\{X^p\}\) and \(\{Y^p\}\) – coordinate vectors of the nodes, having the form:

\[
\{X^p\}^T = [x_2^p \quad x_3^p \quad x_4^p],
\]

\[
\{Y^p\}^T = [y_2^p \quad y_3^p \quad y_4^p].
\]

The matrix \([g^{p-1}]\) will have the following form:

\[
[g^{p-1}] = [g_x^{p-1} \quad g_y^{p-1}],
\]

where \(\{g_x^{p-1}\}\) and \(\{g_y^{p-1}\}\) – vectors of the boundary conditions, having the form:

\[
\{g_x^{p-1}\}^T = [N_{1,2}^{p-1} \cdot x_1 + N_{1,3}^{p-1} \cdot x_1 + N_{3,5}^{p-1} \cdot x_5, N_{4,5}^{p-1} \cdot x_5],
\]

\[
\{g_y^{p-1}\}^T = [N_{1,2}^{p-1} \cdot y_1 + N_{1,3}^{p-1} \cdot y_1 + N_{3,5}^{p-1} \cdot y_5, N_{4,5}^{p-1} \cdot y_5].
\]

The matrix \([\psi^p]\) will look like this:

\[
[\psi^p] = [\psi_x^p \quad \psi_y^p],
\]

where \(\{\psi_x^p\}\) and \(\{\psi_y^p\}\) – vectors of the components of external influences, having the form:
The matrix \([N^{p-1}]\) will look like this:

\[
\begin{bmatrix}
- \left( N_{1,2}^{p-1} + N_{2,3}^{p-1} + N_{2,4}^{p-1} \right) & N_{2,3}^{p-1} & N_{2,4}^{p-1} \\
N_{2,3}^{p-1} & - \left( N_{1,3}^{p-1} + N_{2,3}^{p-1} + N_{3,4}^{p-1} \right) & N_{3,4}^{p-1} \\
N_{2,4}^{p-1} & N_{3,4}^{p-1} & - \left( N_{2,4}^{p-1} + N_{3,4}^{p-1} + N_{4,5}^{p-1} \right)
\end{bmatrix}
\]  

(4.20)

The vector \(\{N^p\}\) will look as shown below:

\[
\{N^p\}^T = \begin{bmatrix}
N_{1,2}^p & N_{1,3}^p & N_{2,3}^p & N_{2,4}^p & N_{3,4}^p & N_{3,5}^p & N_{4,5}^p
\end{bmatrix}.
\]  

(4.21)

The vector \(\{N^{p-1}\}\) has an analogous form, with the exception of the index \(p-1\).

The matrix \([(\delta^p)^2]\) will look like this:

\[
\begin{bmatrix}
\chi \cdot \delta_{1,2}^2 & 0 & \delta_{2,3}^2 & \delta_{2,4}^2 & 0 & 0 & 0 \\
0 & \chi \cdot \delta_{1,3}^2 & \delta_{2,3}^2 & 0 & \delta_{3,4}^2 & \delta_{3,5}^2 & 0 \\
\delta_{1,2}^2 & \delta_{1,3}^2 & \chi \cdot \delta_{2,3}^2 & \delta_{2,4}^2 & \delta_{3,4}^2 & \delta_{3,5}^2 & 0 \\
0 & \delta_{1,3}^2 & \delta_{2,3}^2 & \delta_{2,4}^2 & \chi \cdot \delta_{3,4}^2 & \delta_{3,5}^2 & 0 \\
0 & \delta_{1,3}^2 & \delta_{2,3}^2 & 0 & \delta_{3,4}^2 & \chi \cdot \delta_{3,5}^2 & 0 \\
0 & 0 & 0 & \delta_{2,4}^2 & \delta_{3,4}^2 & 0 & \chi \cdot \delta_{3,5}^2
\end{bmatrix}.
\]  

(4.22)

Taking into account the coefficients \(\delta\) determined by the formula (3.2), the vector \(\{\varphi^p\}\) will have the following form:

\[
\{\varphi^p\} = \begin{bmatrix}
\varphi_{1,2}^p \\
\varphi_{1,3}^p \\
\varphi_{2,3}^p \\
\varphi_{2,4}^p \\
\varphi_{3,4}^p \\
\varphi_{3,5}^p \\
\varphi_{4,5}^p
\end{bmatrix} = \begin{bmatrix}
\partial_{1,2}^p \cdot \varphi_{2}^p \\
\partial_{1,3}^p \cdot \varphi_{3}^p \\
\partial_{2,3}^p \cdot (\varphi_{2}^p + \varphi_{3}^p) \\
\partial_{2,4}^p \cdot (\varphi_{2}^p + \varphi_{4}^p) \\
\partial_{3,4}^p \cdot (\varphi_{3}^p + \varphi_{4}^p) \\
\partial_{3,5}^p \cdot \varphi_{5}^p \\
\partial_{4,5}^p \cdot \varphi_{5}^p
\end{bmatrix}.
\]  

(4.23)

Here, the values of the nodal potentials must be calculated by the formula (3.1).

The matrix of the expected geometric parameters \([(\delta^i)^2]\) will have the form similar to the matrix \([(\delta^p)^2]\), since the position of all nodes after the adjustment will be known. Elements of this matrix will look like:

\[
\delta_{i,j}^2 = (x_{i,j}^p - x_{i,j}^p)^2 + (y_{i,j}^p - y_{i,j}^p)^2.
\]  

(4.24)
It is obvious, that when the truss reaches the expected form, the values of the node target functions are equal to zero. Therefore, the vector of expected values of the potentials \( \{\phi^p\} \) should contain only zero values:

\[
\{\phi^p\} = \begin{bmatrix}
\phi_{1,2}^p \\
\phi_{1,3}^p \\
\phi_{2,3}^p \\
\phi_{2,4}^p \\
\phi_{3,4}^p \\
\phi_{3,5}^p \\
\phi_{4,5}^p
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}.
\] (4.25)

Using the algorithmic formulas (2.5) and (3.11), as well as expressions (4.11) – (4.25), we perform the shaping, adjustments and calculation of the parameters of the truss which are shown in Figure 4, starting from the initial conditions, which are given in Table 1. To simplify the analysis of the results, let’s consider examples only with changes in the heights (coordinates along the axis \( Oy \)) of free nodes.

Nodal forces are represented by vertical and unit: \( F_{y2} = F_{y3} = F_{y4} = -1 \) (kN), \( F_{x2} = F_{x3} = F_{x4} = 0 \) (kN). The accuracy of the absolute error of the iterative calculus for coordinates in all cases is set equal to \( \varepsilon = 0.01 \) (m). The initial values of the conditional stiffness parameters are determined basing on the classical principles of the rod structures, according to which the lower belts most often work for tension, and the upper ones – for compression. Therefore, for the rods of the lower belt, the stiffness parameters must be positive, and the stiffness parameters of the lower rods – must be negative.

Let’s consider all these examples.

**Examples No. 1 and No. 2.** These examples demonstrate the importance of successfully selecting the coefficient \( \chi \) of the parametric equations (4.4) – (4.10). In example No. 1, the value of this coefficient is +0.1. In this case, the number of iterations required to reach the set error value is 200. If the coefficient is increased to +0.2, the number of iteration cycles decreases to 100. Figure 15 shows the truss that was formed basing on the initial conditions No. 1 and No. 2 from the Table 1. The shape of the truss, after adjusting the position of its free nodes is shown in Figure 4.

**Examples No. 3 and No. 4.** Basing on the initial conditions used in examples No. 3 and No. 4, we obtain the original form of the truss, shown in Figure 16. However, after adjusting the truss, the node of its lower belt \( S_3 \) will lie on the same line with the support nodes \( S_1 \) and \( S_5 \) (see Figure 17). To adjust and calculate the parameters of the truss by the initial conditions of examples No. 3 and No. 4, it is necessary to perform 100 and 20 iterative cycles respectively. The coefficients
χ and η in these examples are different. The calculated conditional stiffness parameters are also a little different, which indicates an insufficient value of the established error ε, as well as a significant influence of the initial conditions (coefficients χ and η) on the results and speed of calculations, provided that the construction is symmetric and statically undefined.

**Example No. 5.** This example shows what will be the shape of the truss, if you change all the signs of stiffness parameters of rods (comparatively to examples No. 1 and No. 2) and, accordingly, the character of their work. The truss after shaping is shown in the Figure 18. Obviously, its shape is ineffective, because of all the rods of the truss will be located below the level of the support nodes, and therefore they will occupy a useful volume of internal premises and will be overloaded with snow and other normative loads. In addition, the resulting truss requires the expenditure of excess metal for manufacturing. For amending this, we raise all free nodes up 2 m higher. The shape of this truss after the adjustment is shown in the Figure 19.

![Figure 15. Formation of truss according to the initial conditions No. 1 and No. 2 of Table 1](image)

![Figure 16. Formation of truss according to the initial conditions No. 3 and No. 4 of Table 1](image)
In all shown above examples, we considered statically indeterminate trusses, despite the fact that most often with a single combination of loads it is better to use a simplified statically defined models. It was done to complicate tasks. The changes of construction into a statically determinate would lead to a changes of number and form of the equations system (4.1) – (4.3) and (4.4) – (4.10).
Table 1. The initial conditions and the results of computer calculations

<table>
<thead>
<tr>
<th>Type of data</th>
<th>Variants of initial conditions and results of calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. 1</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>200</td>
</tr>
<tr>
<td>Coefficient $\chi$</td>
<td>+0.1</td>
</tr>
<tr>
<td>Coefficient $\eta$ ($kN \cdot m$)</td>
<td>+0.1</td>
</tr>
<tr>
<td>$x_1$</td>
<td>1.0</td>
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<tr>
<td>$x_2$</td>
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<tr>
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</tr>
<tr>
<td>$y_2$</td>
<td>2.5</td>
</tr>
<tr>
<td>$y_3$</td>
<td>0.5</td>
</tr>
<tr>
<td>$y_4$</td>
<td>2.5</td>
</tr>
<tr>
<td>$y_5$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Given values

<table>
<thead>
<tr>
<th>Expected coordinates of nodes, $m(x_i, y_i)$</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
<th>No. 4</th>
<th>No. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{1,2}$</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>$N_{1,3}$</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$N_{2,3}$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>$N_{2,4}$</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>$N_{3,4}$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>$N_{3,5}$</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$N_{4,5}$</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
</tr>
</tbody>
</table>

Initial conditional stiffness parameters, kN/m ($N_{i,j}$)

<table>
<thead>
<tr>
<th>Calculated values</th>
<th>Variants of initial conditions and results of calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. 1</td>
</tr>
<tr>
<td>$x_1$</td>
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<tr>
<td>$y_1$</td>
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<tr>
<td>$y_2$</td>
<td>2.49250</td>
</tr>
<tr>
<td>$y_3$</td>
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<tr>
<td>$y_4$</td>
<td>2.49250</td>
</tr>
<tr>
<td>$y_5$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Obtained coordinates of nodes, m ($x_i, y_i$)

<table>
<thead>
<tr>
<th>Obtained conditional stiffness parameters, kN/m ($N_i,j$)</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
<th>No. 4</th>
<th>No. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{1,2}$</td>
<td>-0.65432</td>
<td>-0.65769</td>
<td>-1.49589</td>
<td>-1.50034</td>
<td>+1.21640</td>
</tr>
<tr>
<td>$N_{1,3}$</td>
<td>+1.02662</td>
<td>+1.05491</td>
<td>-0.02646</td>
<td>-0.02271</td>
<td>-0.88825</td>
</tr>
<tr>
<td>$N_{2,3}$</td>
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<td>-0.01050</td>
<td>+0.50147</td>
<td>+0.49973</td>
<td>+0.25758</td>
</tr>
<tr>
<td>$N_{2,4}$</td>
<td>-0.32722</td>
<td>-0.31787</td>
<td>-1.00242</td>
<td>-0.99969</td>
<td>+0.48359</td>
</tr>
<tr>
<td>$N_{3,4}$</td>
<td>-0.00627</td>
<td>-0.01050</td>
<td>+0.50147</td>
<td>+0.49973</td>
<td>+0.25758</td>
</tr>
<tr>
<td>$N_{3,5}$</td>
<td>+1.02661</td>
<td>+1.05491</td>
<td>-0.02646</td>
<td>-0.02271</td>
<td>-0.88825</td>
</tr>
<tr>
<td>$N_{4,5}$</td>
<td>-0.65432</td>
<td>-0.65769</td>
<td>-1.49589</td>
<td>-1.50034</td>
<td>+1.21640</td>
</tr>
</tbody>
</table>
5. Conclusions

Thus, the demonstrated mathematical apparatus and the correction technique allow not only to pre-simulate the shape of the rod structure, taking into account the predefined nature of the operation of its elements, but also to adjust its geometrical and physical parameters without using the supplementary numerical methods of calculations. It should be noted that this technique could be attributed to algorithms of systemic optimization, using logical operators and special target functions. The methodology is sufficiently flexible and can be relatively easily modified, since it represents a symbiosis of discrete geometric modeling methods and numerical simulation methods.

The used mathematical apparatus requires further research and additional experimental tasks aimed at identifying the optimal conditions for minimizing the execution time of the iterative calculations. At the same time, the technique is of considerable interest, since it can allow uniting the instrumental bases of architects and design engineers, reducing their losses of working time by means of the complex solution of the problems of shaping and determining the components of the stress-strain state of the rod structures.

However, it is not strictly necessary to use the proposed methodology for complex solving the problems of shaping and adjusting of structures. It is permissible to use additional numerical methods, such as the finite element method, to solve the problem of correction, using results of the formation only. And conversely, the shaping stage can be generally missed, and the initial form of the truss can be accepted as standard or based on constructive considerations. In this case, only the correction algorithm may be used.

References

5. Ploskyi V., Skochko V.: *The Discrete Cages of Surfaces Constructing Using*


